# How to Design a Paraxial Zoom Lens

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A zoom lens is a lens system whose focal length (and thus angle of view) can be varied while the optical power of each lens element in the lens remains unchanged. Ideally, a zoom lens should be parfocal, i.e. it maintains focus when its focal length changes. Most consumer-grade zoom lenses do not maintain perfect focus, although they are nearly parfocal. Note that most camera phones do not have optical zoom but instead use several cameras of different but fixed focal length combined with digital zoom.

The most commonly adopted structure for zoom lens is shown in Figure 1, where the first three lenses, of focal length $f_1$, $f_2$, and $f_3$, and optical power $K_1 = 1/f_1$, $K_2 = 1/f_2$, and $K_3 = 1/f_3$, forms a telescope with the object and image located at negative and positive infinity, respectively. The last lens then focuses the image from the third lens on to the final image plane IP. The change in the focal length is achieved through the distances $d_1$ and $d_2$ while maintaining the telescope condition, $h_1K_1 + h_2K_2 + h_3K_3 = 0$, which ensures that the focus on the image plane does not change.

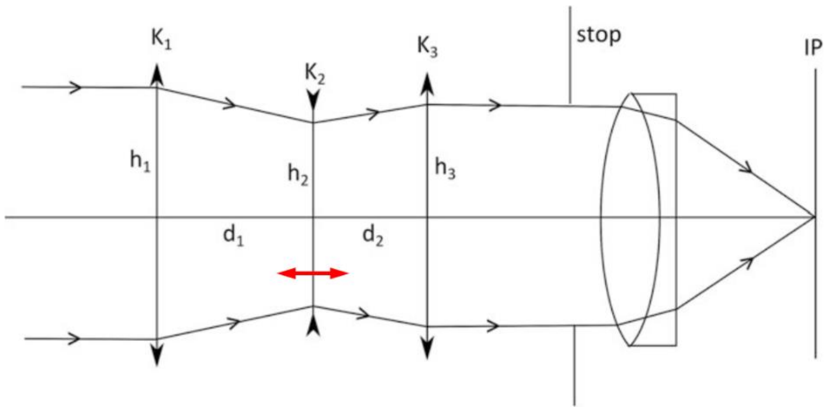


Figure 1. A typical structure of zoom lens.

To analyze the relationship between the distances and the zoom range, we consider that the marginal ray height and angle at the first lens are $h_1$ and $u_1$, respectively, where $u_1 = 0$. At the second lens, then, the marginal ray height and angle are $h_2$ and $u_2$, respectively, where $u_2 = -h_1K_1$. Finally, at the third lens, the marginal ray height and angle are $h_3$ and $u_3$, respectively, where $u_3 = -h_3K_3$. These relationships lead to the *angular* magnification as

$$M = \frac{h_1}{h_3} = \frac{-K_3}{K_1 + K_2 - d_1K_1K_2}.$$

The job of the design process is then to find out how to change $d_1$ to achieve the range of magnification required. As we have seen in the class, $d_2$ must be change in coordination with $d_1$ to maintain the telescope condition. For convenience of analysis, we now assume that $K_2 = -pK_1$, $K_3 = qK_1$, $d_1 = af_1$, and $d_2 = bf_1$, where $p$, $q$, $a$, and $b$ are (positive) coefficients. The magnification is then

$$M = \frac{-q}{1 + p(a-1)},$$

where we note that $a$ must satisfy $0 < a < 1 - \frac{1}{p}$.

Given fixed $K_1$, $K_2$, and $K_3$, and thus $p$ and $q$, what we need to find is $M$ in relation to $p$ and $q$. To do so, we note that at second lens receives an image from the first lens as the object with an object distance $s = f_1 - d_1$, which results in an image distance

$$s' = \frac{1}{-pK + \frac{1}{f_1 - d_1}} = \frac{1-a}{1 + p(a-1)} f_1 .$$

To satisfy the telescope condition, this image by the second lens must be the object located at the object side focus of the third lens. Therefore, we have

$$d_2 = bf_1 = \frac{f_1}{q} - s' = \frac{p(a-1) + q(a-1) + 1}{pq(a-1) + q} f_1 ,$$

which leads to

$$b = \frac{p(a-1) + q(a-1) + 1}{pq(a-1) + q}.$$

As an example, let's consider a specific case where $p = 2$, $q = 1$, and plot $M$ and $b$ as a function of $a$ in Figure 2. In a design with a target zoom ratio, one needs to fix a choice of $p$ and $q$ to solve the equation for the range of $a$ and subsequently $b$ as a function of $a$. In a physical implementation, the coordinated movements to change $a$ and $b$ are realized through a complex mechanical means or a microcontroller embedded in the lens or camera.

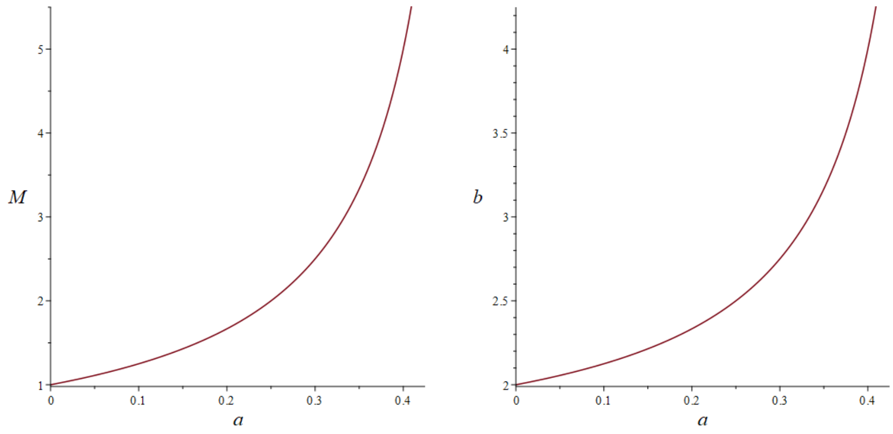


Figure 2. *M* and *b* as a function of *a*.